

Enrollment No: \_\_\_\_\_ Exam Seat No: \_\_\_\_\_

# C.U.SHAH UNIVERSITY

## Winter Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2

Branch: B. Tech (All)

Semester : 2

Date : 23/10/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1 Attempt the following questions: (14)**

a) If  $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta d\theta$ , then for any positive integer n, the value of  $n(I_{n-1} + I_{n+1})$

is

(A) 1 (B) 2 (C)  $\frac{\pi}{4}$  (D)  $\pi$

b) Find the value of  $\int_{-1}^1 \sin^{11} x dx$

(A) 10! (B)  $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{5} \cdot \frac{\pi}{2}$  (C) 0 (D) none of these

c)  $\sqrt{4.5} =$  \_\_\_\_\_

(A)  $\frac{\sqrt{\pi}}{16}$  (B)  $\frac{105\sqrt{\pi}}{16}$  (C)  $\frac{5\sqrt{\pi}}{16}$  (D) none of these

d) If  $B(x, 2) = \frac{1}{3}$ , then the value of  $x =$  \_\_\_\_\_.

(A) 0 (B) 1 (C) 2 (D) none of these

e)  $\int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{1}{4} \sin^2 \theta} d\theta$  is equal to

(A)  $E\left(\frac{1}{2}\right)$  (B)  $E\left(\frac{1}{4}\right)$  (C)  $K\left(\frac{1}{2}\right)$  (D)  $K\left(\frac{1}{4}\right)$

f)  $\int_{-a}^a e^{-t^2} dt$  is equal to



(A)  $\sqrt{\pi} \operatorname{erf}(a)$  (B)  $\sqrt{\pi} \operatorname{erfc}(a)$  (C)  $\frac{\sqrt{\pi}}{2} \operatorname{erf}(a)$  (D)  $\frac{\sqrt{\pi}}{2} \operatorname{erfc}(a)$

g) If the power of  $y$  are even, then the curve is symmetrical about  
 (A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these

h) The curve  $y^2(a+x) = x^2(a-x)$  where  $a > 0$  represent  
 (A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid  
 (D) Folium of Descartes

i)  $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx dy$  is equal to

(A)  $\pi a^2$  (B)  $\frac{\pi a^2}{2}$  (C)  $\frac{\pi a^2}{4}$  (D) none of these

j) The transformations  $x+y=u, y=uv$  transform the area element  $dy dx$  into  $|J| du dv$ , where  $|J|$  is equal to  
 (A) 1 (B)  $u$  (C)  $-1$  (D) none of these

k) The degree of the differential equation  $3 \frac{d^2 y}{dx^2} = \left\{ 1 + \left( \frac{dy}{dx} \right)^2 \right\}^{\frac{3}{2}}$  is

(A) 1 (B) 2 (C) 3 (D) 6

l) If  $\frac{dy}{dx} + \frac{1}{y\sqrt{1-x^2}} = 0$ , then which of the following statements is true?

(A)  $y + \sin^{-1} x = 0$  (B)  $y^2 + 2\sin^{-1} x = c$  (C)  $x + \sin^{-1} y = c$  (D)  $y = k$

m) The series  $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \dots \infty$  is

(A) convergent (B) divergent (C) finitely oscillating (D) infinitely oscillating

n) The sum of the series  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$  is

(A)  $\log 2$  (B) zero (C) infinite (D) none of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

a) Using reduction formula evaluate:  $\int_0^1 x^6 \sin^{-1} x dx$  (5)

b) Prove that  $\int_{-\infty}^{\infty} e^{-k^2 x^2} dx = \frac{\sqrt{\pi}}{k}$  (5)

c) Evaluate:  $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} z dx dy dz$  (4)

**Q-3 Attempt all questions (14)**



a) Show that  $\int_0^2 x^4 (8-x^3)^{-\frac{1}{3}} dx = \frac{16}{3} B\left(\frac{5}{3}, \frac{2}{3}\right)$  (5)

b) Solve:  $\frac{dy}{dx} = 2y \tan x + y^2 \tan^2 x$  (5)

c) Discuss the convergence of  $\sum \frac{\sqrt{n+1} - \sqrt{n}}{n}$ . (4)

**Q-4 Attempt all questions** (14)

a) Change the order of integration in the integral  $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$  and evaluate it. (5)

b) Test the convergence of the series  $\frac{1}{1 \cdot 2 \cdot 3} + \frac{x}{4 \cdot 5 \cdot 6} + \frac{x^2}{7 \cdot 8 \cdot 9} + \dots$  (5)

c) Evaluate:  $\int_0^{\frac{1}{2}} x^3 \sqrt{1-4x^2} dx$  (4)

**Q-5 Attempt all questions** (14)

a) Solve:  $x dy - y dx = x \sqrt{x^2 - y^2} dx$  (5)

b) By changing into polar co-ordinates, evaluate the integral (5)

$$\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx dy$$

c) Prove that  $B(m, n)B(m+n, p)B(m+n+p, q) = \frac{\Gamma(m) \Gamma(n) \Gamma(p) \Gamma(q)}{\Gamma(m+n+p+q)}$  (4)

**Q-6 Attempt all questions** (14)

a) Show that  $\sum_{n=1}^\infty \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$  is (i) convergent if  $p > 1$  and (ii) divergent if  $p \leq 1$ . (5)

b) Using reduction formula prove that  $\int_0^1 \frac{x^6}{1+x^2} dx = \frac{13}{15} - \frac{\pi}{4}$ . (5)

c) Solve:  $(x^2 + y^2 - a^2)x dx + (x^2 - y^2 - b^2)y dy = 0$  (4)

**Q-7 Attempt all questions** (14)

a) Trace the curve  $xy^2 = 4a^2(2a-x)$ . (5)

b) Find the area of the loop of the curve  $x^3 + y^3 = 3axy$ . (5)

c) Evaluate:  $\int_1^\infty \frac{dx}{\sqrt{x^4-1}}$  (4)

**Q-8 Attempt all questions** (14)

a) Prove that  $erf_c(x) + erf_c(-x) = 2$ . (5)

b) Find the asymptotes of the curve  $y^3 - x^2(6-x) = 0$ . (5)

c) Find the perimeter of the cardioids  $r = a(1 + \cos \theta)$ . (4)

